Ordinal Evaluation And Assignment Problems*

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***Abstract*—In many assignment problems, a set of documents such as research proposals, promotion dossiers, resumes of job applicants is assigned to a set of experts for ordinal evaluation, ranking, and classification. A desirable condition for such assignments is that every pair of documents is compared and ordered by one or more experts. This condition was modeled as an optimization problem and the number of pairs of documents was maximized for a given incidence relation between a set of documents and a set of experts using a set covering integer programming method in the literature[5]. In this paper, we use a combinatorial approach to derive lower bounds on the number of experts needed to compare all pairs of documents and describe assignments that asymptotically match these bounds. These results are not only theoretically interesting but also have practical implications in obtaining optimal assignments without using complex optimization techniques.**

***Keywords*—assignment problems, combinatorial assignment, document evaluation, ordinal ranking, peer review.**

I. INTRODUCTION

Ordinal document evaluation and ranking arises in several contexts that make use of collective or committee-based evaluation and ranking systems [2,4,8,10,12,15]. Informally speaking, the evaluation of documents consists of two interrelated tasks: (a) assignment of documents to a set of experts, (b) ranking and selection of documents. The ranking and selection of documents typically relies on cardinal (quantitative) or ordinal (preference) –based comparisons as described in the context of research documents in [6,13]. Cook et al. demonstrated that cardinal comparisons such as using average scores of documents could be unreliable especially when experts' scores are not normalized [5-7]. They suggested that quantifying the intrinsic values of documents may be difficult, and therefore it is more practical to rely on ordinal rankings. A set-covering integer programming approach was introduced in [5] to obtain as many comparisons as possible between the documents reviewed by a fixed set of experts. More recently, a maximum consensus algorithm based on complete rankings of a set of documents by a set of experts was presented in [3], and dominance-based ordinal ranking and selection algorithms were presented in [1,9,14].

This paper is concerned with the assignment aspect of ordinal evaluations of documents that can be research proposals, promotion dossiers of academic personnel, resumes of job applicants, etc. In ordinal ranking, a limited coupling between such documents and experts may potentially divide documents into disjoint clusters and make it impossible to compare documents between clusters. The approach described in [5] fixes documents and experts together with an incidence relation to specify which documents can potentially be assigned to which experts.

In this paper, we consider assignment problems with only two parameters of interest: the number of documents, n, and the capacity of each expert, k, $2 \leq k \leq n$, i.e., the maximum number of documents that can be reviewed by each expert. With these two parameters, we consider two related problems: (1) determine the minimum number of experts that ensures that each pair of documents is reviewed by at least one expert, (2) find an assignment of a set of n documents to the minimum number of experts as determined in (1) so that all pairs of documents are reviewed by at least one expert. Our interest in these problems is motivated by the fact that experts are generally selected to meet the evaluation needs of a set of documents rather than randomly assembled together. Thus, unlike in the assignment problems considered in [5,7], minimizing the number of experts is the main objective in the assignments of documents to experts in our work. We consider the assignments of documents to experts without specialties. That is, we assume that experts are interchangeable in terms of their expertise. This assumption generally holds for those document evaluation processes in which a small set of documents with identical topics is considered, or for those in which a large set of documents is pre-screened to identify a small set of documents for a second stage of a more intense review. In other document evaluation problems, experts with specialties may be required as in peer review panels in which documents with a multitude of subjects are put together and evaluated. The assignments described in this paper can be modified and applied to assignment problems that require experts with different specialties [16].

The main contributions of the paper are (1) a general lower bound for the number of experts when the capacity of each expert is fixed to k, $2 \leq k \leq n$, (2) tighter lower bounds when the capacity is fixed to $n/2$, $n/3$, and $n/4$, (3) an assignment that matches the lower bound when the capacity of each expert is $n/2$, and assignments that nearly match the lower bounds when the capacity of each expert is $n/3$ and $n/4$, and (4) an assignment that matches the general lower bound within a factor of 2 for any $k \leq n/2$ that divides n.

The rest of the paper is organized as follows. Section II presents our lower bounds for the number of experts, and in Section III, our assignments for the three expert capacities as mentioned above are presented. The assignment for the arbitrary expert capacity case is given in Section IV. The paper is concluded in Section V.

II. LOWER BOUNDS

Let $D = \{d_1, d_2,\ldots,d_n\}$ be a set of documents, $n \geq 2$, and let $E = \{e_1, e_2,\ldots,e_m\}$ be a set of experts. The experts in E are said to cover all $n(n-1)/2$ pairs of n documents if each pair of documents is reviewed by at least one expert in E. Suppose that

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each expert is willing to review k documents, where k, $2 \leq k \leq n$. Then, for all $n(n-1)/2$ pairs of documents to be covered by the m experts, the following inequality must clearly hold:

$$m\binom{k}{2} \geq \binom{n}{2}, \quad k \geq 2 \qquad (1)$$

Simplifying this inequality gives the following lower bound on the number of experts:

$$m = \left\lceil \frac{n(n-1)}{k(k-1)} \right\rceil, \quad k \geq 2 \qquad (2)$$

In particular, when $k = 2$, i.e., when each expert reviews 2 documents, a minimum of $n(n-1)/2$ experts is required, and when $k = n$, one expert is required. Other constraints can be derived from this inequality. Table I lists the capacities of experts versus the minimum numbers of experts for various values of n. It is obvious that when $k = n$, and $n \geq 2$, one expert will suffice, and hence $m = 1$ is always achievable.

TABLE I. Minimum number of experts for certain capacities.

	Minimum Number of experts (m)			
Capacity	Eqn. (2)	$n = 12$	$n = 24$	$n \to \infty$
$k = n$	$m \geq 1$	$k = 12, m \geq 1$	$k = 24, m \geq 1$	$m \to 1$
$k = n/2$	$m \geq \lceil 4(n-1)/(n-2) \rceil$	$k = 6, m \geq 5$	$k = 12, m \geq 5$	$m \to 5$
$k = n/3$	$m \geq \lceil 9(n-1)/(n-3) \rceil$	$k = 4, m \geq 11$	$k = 8, m \geq 10$	$m \to 10$
$k = n/4$	$m \geq \lceil 16(n-1)/(n-4) \rceil$	$k = 3, m \geq 22$	$k = 6, m \geq 19$	$m \to 17$

For even n and $k = n/2$, the table shows that m tends to 5 as $n \to \infty$. However, for $n = 4$, Eqn. (2) implies that $m = 6$. We strengthen the lower bound to 6 for other values of n as follows.

Theorem 1: For all even $n = 2k \geq 4$, if each expert is assigned k documents, at least 6 experts are needed to cover all pairs of n documents.

Proof: For $n = 4$, $k = 2$, each expert is assigned two documents, and can therefore cover only one pair. Since there are 6 pairs of documents in all, 6 experts are clearly necessary. For any even $n \geq 6$, without loss of generality, suppose that the first two experts are assigned k documents as shown below with u documents shared between them, where u is an integer between 0 and k, and the shaded areas represent the sets of documents assigned to the two experts:

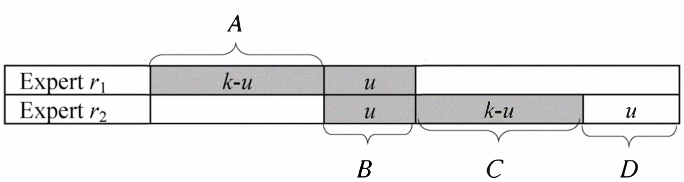

Figure 1.

Then we have the following sets of pairs of documents that remain to be covered:

$$A \times C = \{(a,c) : a \in A,\ c \in C\}$$

$$A \times D = \{(a,d) : a \in A,\ d \in D\}$$

$$B \times D = \{(b,d) : b \in B,\ d \in D\}$$

$$C \times D = \{(c,d) : c \in C,\ d \in D\}$$

$$D \times D = \{(d_1,d_2) : d_1,d_2 \in D, d_1 < d_2\}$$

If $u = 0$ then B and D vanish, and $|A| = |C| = k$ so that the number of additional pairs of documents that remain to be covered is given by k^2. Furthermore, in order to cover these k^2 pairs of documents, each additional expert must be assigned at least one document from each of A and C. Therefore, the number of additional experts cannot be less than

$$\left\lceil \frac{k^2}{w(k-w)} \right\rceil,$$

where w denotes the number of documents in A and k-w denotes the number of documents in C. Since the denominator is maximized when $w = k/2$, the number of additional experts cannot be less than 4 implying that 6 experts are necessary in this case.

On the other hand, if $u = k$ then A and C vanish, and $|B| = |D| = k$ so that the number of additional pairs of documents to be covered is given by $k^2+k(k-1)/2$. But since each new expert can cover at most $k(k-1)/2$ documents, we need at least

$$\left\lceil \frac{k^2 + k(k-1)/2}{k(k-1)/2} \right\rceil = \left\lceil \frac{3k^2 - k}{k^2 - k} \right\rceil \geq 4, \text{ for } k > 1$$

more experts[1]. Therefore, at least 6 experts are needed to cover all pairs of n documents in this case as well.

To complete the proof, suppose that $1 \leq u < k$. In this case, we must cover the pairs of documents in all the sets stated above. In particular, we must cover the pairs of documents in the sets $A \times C$, $A \times D$, $B \times D$, and $C \times D$. This leads to the assignment pattern for the additional experts as follows:

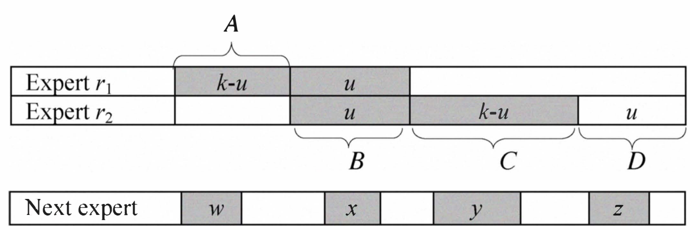

Figure 2.

Therefore, the number of additional experts cannot be less than

$$\left\lceil \frac{(k-u)(k-u+u) + (u+k-u)u}{wy + wz + xz + yz} \right\rceil = \left\lceil \frac{k^2}{wy + wz + xz + yz} \right\rceil$$

where w, x, y, z are the numbers of documents assigned to a new expert from the subsets, A, B, C, and D, respectively. It can be shown that, under the constraint $w + x + y + z = k$, the denominator of the expression has a unique global maximum at $x = 0$, $w = y = z = k/3$, and is given by $k^2/3$. However, since $u \geq 1$, the value of x cannot be zero for all additional experts as this will leave out one or more pairs of documents one of which belongs to B. Therefore, the maximum number of pairs generated by at least one of the additional experts must be less than $k^2/3$, and hence the number of additional experts cannot be less than 4. Adding these to the first two experts shows that 6 experts are necessary in this case as well and this completes the proof. ||

1 $\frac{3k^2-k}{k^2-k} > 3$ for all $k > 1$. Therefore, $\left\lceil \frac{3k^2-k}{k^2-k} \right\rceil \geq 4$ for all $k > 1$.

Corollary 1: For all odd $n = 2k+1 \geq 5$, suppose that at most three experts can be assigned at most $k+1$ documents each, and other experts can be assigned at most k documents each. Then at least 6 experts are needed to cover all pairs of n documents.

Proof: Let $n = 2k+1$, where $k \geq 2$. Consider any $2k$ of the n documents, and let d be the document that is left out. By Theorem 1, at least six experts must be used, with each assigned k documents, to cover all $2k(2k-1)/2 = k(2k-1)$ pairs of these $2k$ documents. This leaves

$$(2k+1)2k/2 - k(2k-1) = k\{(2k+1) - (2k-1)\} = 2k$$

pairs of documents still to be covered. Suppose that one of the experts is removed and document d is assigned to three of the remaining five experts each, in addition to their k documents which they had been originally assigned. Now, with one of the experts removed, at least one pair of documents among the first $2k$ documents, previously covered by the six experts must clearly be left uncovered. Otherwise, five experts would have been sufficient to cover the original $2k$ documents. Therefore, at least $2k+1$ pairs of documents must now be covered by the three experts whose assignments have been increased by one document. However, with one new document, i.e., document d, these three experts can collectively increase the number of pairs of documents by at most $2k$ since the three experts were assigned their k documents from the original set of $2k$ documents prior to the assignment of document d. But, this is less than the $2k+1$ pairs of documents still to be covered and the statement follows. ||

The following two theorems extend these results to experts with capacities of $n/3$ and $n/4$ and tighten the lower bounds given in Eqn. (1) by one. The reader is referred to [16] for proofs.

Theorem 2: For all $n = 3k \geq 12$, if each expert is assigned k documents then at least 11 experts are needed to cover all pairs of n documents[2]. ||

Theorem 3: For all $n = 4k \geq 16$, if each expert is assigned k documents, at least 18 experts must be used to cover all pairs of n documents. ||

III. ASSIGNMENTS-UPPER BOUNDS

We first present an optimal assignment of n documents to experts with a capacity of about $n/2$.

Theorem 4:

(a) For any even integer $n = 2k \geq 4$, if four experts are assigned k documents each, one expert is assigned $2\lceil k/2\rceil$ documents and one expert is assigned $2\lfloor k/2\rfloor$ documents, then six experts are sufficient to cover all pairs of n documents.

(b) For any odd integer $n = 2k+1 > 5$, if one half of experts are assigned $\lceil n/2\rceil$ documents each, and the other half of experts are assigned $\lfloor n/2\rfloor$ documents each then six experts are sufficient to cover all pairs of n documents.

[2] The statement can be extended to $n = 3k+1$ and $n = 3k+2$ using a similar argument as in Corollary 1.

Table II. Assignment of $n = 2k$ documents to six experts, each with a capacity of k.

Documents	$d_1, \ldots, d_{\lceil k/2\rceil}$	$d_{\lceil k/2\rceil+1}, \ldots, d_k$	$d_{k+1}, \ldots, d_{k+\lceil k/2\rceil}$	$d_{k+\lceil k/2\rceil+1}, \ldots, d_{2k}$
Expert e_1	k documents			
Expert e_2			k documents	
Expert e_3	$\lceil k/2\rceil$		$\lceil k/2\rceil$	
Expert e_4	$\lceil k/2\rceil$			$\lfloor k/2\rfloor$
Expert e_5		$\lfloor k/2\rfloor$	$\lceil k/2\rceil$	
Expert e_6		$\lfloor k/2\rfloor$		$\lfloor k/2\rfloor$

Proof: (a) For even n, we give one possible assignment that uses six experts as shown in Table II. That this assignment covers all $n(n-1)/2$ pairs of documents can be seen as follows. The first expert covers the $k(k-1)/2$ pairs of the first k documents and the second expert covers the $k(k-1)/2$ pairs of the second k documents, and they are disjoint. The third expert covers $\lceil k/2\rceil\times\lceil k/2\rceil$ pairs of documents and clearly, these pairs are all different from those covered by the first two experts. Likewise, the fourth, fifth, and sixth experts, cover $\lceil k/2\rceil\times\lfloor k/2\rfloor$, $\lfloor k/2\rfloor\times\lceil k/2\rceil$, $\lfloor k/2\rfloor\times\lfloor k/2\rfloor$ pairs of documents which are all distinct from one another and those covered by the first three experts. Hence, the number of pairs covered by the 6 experts is given by

$$2k(k-1)/2 + \left\lceil\frac{k}{2}\right\rceil\times\left\lceil\frac{k}{2}\right\rceil + \left\lceil\frac{k}{2}\right\rceil\times\left\lfloor\frac{k}{2}\right\rfloor + \left\lfloor\frac{k}{2}\right\rfloor\times\left\lceil\frac{k}{2}\right\rceil + \left\lfloor\frac{k}{2}\right\rfloor\times\left\lfloor\frac{k}{2}\right\rfloor$$

$$= k(k-1) + \left\lceil\frac{k}{2}\right\rceil\times\left\{\left\lceil\frac{k}{2}\right\rceil + \left\lfloor\frac{k}{2}\right\rfloor\right\} + \left\lfloor\frac{k}{2}\right\rfloor\times\left\{\left\lceil\frac{k}{2}\right\rceil + \left\lfloor\frac{k}{2}\right\rfloor\right\} = k(k-1) + \left\lceil\frac{k}{2}\right\rceil\times k + \left\lfloor\frac{k}{2}\right\rfloor\times k$$

$$= k(k-1) + \left\{\left\lceil\frac{k}{2}\right\rceil + \left\lfloor\frac{k}{2}\right\rfloor\right\}\times k = k(k-1) + k^2 = k(2k-1) = \binom{2k}{2} = \binom{n}{2}$$

as desired.

(b) The proof for odd n is similar and omitted. ||

Example 1:

(a) $n = 6$, $k = 3$: Covering 6 documents with 6 experts.

e_1	d_1	d_2	d_3			
e_2				d_4	d_5	d_6
e_3	d_1	d_2		d_4	d_5	
e_4	d_1	d_2				d_6
e_5			d_3	d_4	d_5	
e_6			d_3			d_6

e_i denotes expert i.

d_i denotes document i.

capacity = 3.

(b) $n = 8$, $k = 4$: Covering 8 documents with 6 experts.

e_1	d_1	d_2	d_3	d_4				
e_2					d_5	d_6	d_7	d_8
e_3	d_1	d_2			d_5	d_6		
e_4	d_1	d_2					d_7	d_8
e_5			d_3	d_4	d_5	d_6		
e_6			d_3	d_4			d_7	d_8

e_i denotes expert i.

d_i denotes document i.

capacity = 4.

Theorem 5: Suppose that n is divisible by 9, and let $k = n/3$. Then twelve experts are sufficient to cover all pairs of n documents.

Proof: Divide the set of n documents into three groups of k documents each, and use a different expert to review the k documents in each group. This covers $3k(k-1)/2$ pairs of documents with 3 experts and

$$\binom{3k}{2} - 3k(k-1)/2 = 3k^2$$

pairs of documents remain. Now, divide each group of k documents into 3 disjoint groups of $k/3$ documents (see Table III)

$$\mathcal{H}_i = \{G_{i,1}, G_{i,2}, G_{i,3}\}, i = 1,2,3$$

where $|G_{i,j}| = k/3$, $1 \le i, j \le 3$, and use nine experts to cover the remaining pairs of documents as follows: Assign the subsets of $k/3$ documents across H_1, H_2, and H_3 to nine experts in such a way that (1) each expert is assigned exactly one subset of $k/3$ documents in each H_i, and (2) no two experts are assigned the same two subsets of $k/3$ documents from any two different groups, H_i and H_j, $1 \le i \ne j \le 3$. This ensures that the pairs of documents covered by the experts will be distinct. Moreover, each expert is assigned exactly k documents, one subgroup of $k/3$ documents from each of the inner three groups. That this can always be done is proved in [16] and illustrated in Table IV at the end of the paper. To complete the proof, it is sufficient to note that

$$3^2\binom{3}{2}\left(\frac{k}{3}\right)^2 = 3k^2$$

pairs of documents are covered by the nine experts, and adding it to the number of documents covered by the first three experts gives

$$3k(k-1)/2 + 3k^2 = \binom{3k}{2} = \binom{n}{2}$$

pairs of documents as desired. ||

Theorem 6: Suppose that n is divisible by 16, and let $k = n/4$. Then twenty experts are sufficient to cover all pairs of n documents.

Proof: See [16]. ||

The assignments described in Theorems 4, 5, and 6 will work effectively for small values of n. In particular, 6-expert assignments in Theorem 4 can handle up to 20 documents where each expert may be assigned up to 10 documents. However, for larger n, it will be impractical for experts to review $n/2$, $n/3$, or $n/4$ documents, and the number of documents assigned to each expert may have to be decreased as needed. To deal with larger numbers of documents, we present another assignment using an asymptotically minimum number of experts. The following theorem describes this assignment for any even k that divides n. The theorem is easily extended to odd k and omitted here.

Theorem 7: Let n and k be positive integers, where k is even and divides n. It is sufficient to have $n(2n-k)/k^2$ experts, each with capacity k to cover all $n(n-1)/2$ pairs of n documents.

TABLE III. Assignment of n documents to 3+9 = 12 experts.

$n = 3k$ documents

	$\mathcal{H}_1$			$\mathcal{H}_2$			$\mathcal{H}_3$		
Expert e_1	k								
Expert e_2				k					
Expert e_3							k		
Expert e_4	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$
Expert e_5	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$
⋮									
Expert e_{12}	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$	$k/3$
	$G_{1,1}$	$G_{1,2}$	$G_{1,3}$	$G_{2,1}$	$G_{2,2}$	$G_{2,3}$	$G_{3,1}$	$G_{3,2}$	$G_{3,3}$

Proof: Divide the set of n documents into n/k groups, and use a different expert to review the k documents in each group. This covers $n\binom{k}{2}/k$ pairs with n/k experts. Now, use four more experts to cover the pairs of documents between every two groups of k documents as shown in Table V for one such pair of groups. This gives

$$4\binom{n/k}{2}\frac{k^2}{4} = \binom{n/k}{2}k^2$$

more distinct pairs, making the total number of pairs equal to

$$\frac{n}{k}\binom{k}{2} + \binom{n/k}{2}k^2 = \frac{n(k-1)}{2} + \frac{n(n-k)}{2} = \frac{n(n-1)}{2} = \binom{n}{2}$$

as desired. Since there are $\binom{n/k}{2}$ such pairs of groups, the number of experts we need to cover the pairs of documents generated by these pairs of groups is given by $4\binom{n/k}{2}$.

Therefore, the total number of experts to cover all $n(n-1)/2$ pairs of documents is given by

$$\frac{n}{k} + 4\binom{n/k}{2} = \frac{n}{k} + 2\frac{n}{k}\left(\frac{n}{k} - 1\right) = \frac{n(2n-k)}{k^2}$$

and the statement follows. ||

TABLE V. Assignment of n documents to $n(2n-k)/k^2$ experts, each with capacity k.

e_1	k							
e_2		k						
e_3			k					
⋮					…			
$e_{n/k-1}$						k		
$e_{n/k}$								k
$e_{n/k+1}$			$k/2$			$k/2$		
$e_{n/k+2}$			$k/2$				$k/2$	
$e_{n/k+3}$				$k/2$		$k/2$		
$e_{n/k+4}$				$k/2$			$k/2$	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

n documents

Corollary 2: The number of experts used in the assignment described in Theorem 7 is within a factor of 2 of the lower bound given in Section II and therefore is asymptotically optimal.

Proof: Dividing the number of experts obtained in Theorem 7 by the lower bound on the number of experts given in Section II, we obtain

$$\frac{n(2n-k)}{k^2} \times \frac{k(k-1)}{n(n-1)} = \frac{(2n-k)}{k} \times \frac{(k-1)}{(n-1)} < \frac{(2n-k)}{n-1} \le 2, \text{ for } k \ge 2$$

and the statement follows. ||

Remark: For odd k, partition the n documents into n/k groups of k documents each as in Theorem 7 and assign each group to a different expert. Assign $k+1$ documents to each of the rest of experts and divide each group of k documents into two overlapping groups of $(k+1)/2$ documents as in the example below. The rest of the proof applies as it is.

Example 2 (Odd k**):** Let $n = 6$ and $k = 3$. By Eqn. (2), five experts are necessary and by Theorem 7, $n(2n\text{-}k)/k^2 = 6$ experts are sufficient as shown in Table VI(a). As seen in the table, the documents assigned to experts e_3, e_4, e_5, and e_6 overlap. This results in some of the pairs of documents to be covered more than once but it does not increase the number of experts in the assignment. However, it also makes the assignment asymmetric with respect to the number of experts assigned to the documents (documents d_2 and d_5 are reviewed by five experts whereas the rest of documents are reviewed by three experts each). This can be avoided by removing the last expert, and reassigning the documents to remaining experts as shown in Table VI(b). Also, note that this assignment does not contradict Theorem 1 since not all of the experts are assigned $n/2$ =3 documents. ||

Table VI(a). Assignment of 6 documents to 6 experts.

Exp. e_1	d_1	d_2	d_3			
Exp. e_2				d_4	d_5	d_6
Exp. e_3	d_1	d_2		d_4	d_5	
Exp. e_4	d_1	d_2			d_5	d_6
Exp. e_5		d_2	d_3	d_4	d_5	
Exp. e_6		d_2	d_3		d_5	d_6

Table VI(b). Assignment of 6 documents to 5 experts.

Exp. e_1	d_1	d_2	d_3			
Exp. e_2				d_4	d_5	d_6
Exp. e_3	d_1		d_3	d_4	d_5	
Exp. e_4	d_1	d_2			d_5	d_6
Exp. e_5		d_2	d_3	d_4		d_6

IV. CONCLUDING REMARKS

We presented lower bounds on the number of experts with a prescribed capacity needed to cover all pairs of documents by one or more experts. We described an assignment that covers all pairs of n documents and matches the lower bound when the capacity of each expert is equal to $n/2$. We described two other assignments; one that covers all pairs of n documents using one more than the minimum number of experts feasible (12 rather than 11) when the capacity of each expert is equal to $n/3$, and the other that uses two more than the minimum number of experts (20 rather than 18) when the capacity of each expert is equal to $n/4$. Finally, we derived a general lower bound on the number of experts for any expert capacity and described an assignment that matches this lower bound within a factor of 2.

Our results have been stated with certain constraints on the number of documents. For example, we require that n (number of documents) be divisible by 9 in Theorem 5. If the actual number of documents is not divisible by 9, up to 8 dummy documents can be added to make the assignment given in the theorem to work. The only implication of this modification will be that some experts will be assigned one more document than others when the dummy documents are discounted. Theorem 6 can similarly be modified.

Also, we have not considered assignments where experts may be assigned more than $n/2$ documents each. This is explicitly assumed in most of our results except in Theorem 7, in which case, we require k (capacity of experts) to divide n. Clearly no capacity greater than $n/2$ can evenly divide n. Assignments with expert capacities exceeding $n/2$ will reported elsewhere.

We further note that the results presented in this paper can be extended to ordinal covering assignment problems where documents need to be reviewed by experts with specialties. We state one such result here:

Corollary 3: Suppose that a set of n documents can be partitioned into two specialty areas of $n/2$ documents, D_1 and D_2. Further suppose that, among some 6 experts, (a) one is able to review the documents in D_1 and another is able to review the documents in D_2, and (b) each of the other four experts is able to review $n/4$ documents in each of D_1 and D_2. Then all pairs of n documents can be covered by the 6 experts with the side condition that each document is reviewed by three experts in its subject area. ||

The proof follows directly from the assignment table below, where

$$D_1 = \{d_1, d_2, \ldots, d_{n/2}\} \text{ and } D_2 = \{d_{n/2+1}, d_{n/2+2}, \ldots, d_n\}:$$

Documents	$d_1, d_2, \ldots, d_{n/4}$	$d_{n/4+1}, \ldots, d_{n/2}$	$d_{n/2+1}, \ldots, d_{3n/4}$	$d_{3n/4+1}, \ldots, d_n$
Expert 1	Specialty S_1			
Expert 2			Specialty S_2	
Expert 3	Specialty S_1		Specialty S_2	
Expert 4	Specialty S_1			Specialty S_2
Expert 5		Specialty S_1	Specialty S_2	
Expert 6		Specialty S_1		Specialty S_2

We note that the lower bounds on ordinal covering assignments without specialty constraints also apply to those with specialty constraints. Whether these lower bounds can be tightened and/or the upper bounds and assignments given in the paper would still apply under other specialty constraints remain open. Investigation of this and other related problems will be deferred to another place.

A more comprehensive account of the results presented in this paper with proofs and other extensions can be found in [16]. Other combinatorial assignments using balanced incomplete block designs (BIBDs) are also described in [17].

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References

[1] B. S. Ahn and K. S. Park. Comparing methods for multiattribute decision making with ordinal weights. Computers and Operations Research. Vol. 35, no. 5, pp. 1660–1670, May 2008.

[2] L. Bornmann, M. Ruediger, D. Hans-Dieter. Row-column (RC) association model applied to grant peer review. Scientometrics, 73(2), pp. 139–147, 2007.

[3] Y.L. Chen and Li-C. Cheng. Mining maximum consensus sequences from group ranking data. European Journal of Operational Research, 198, pp. 241–251, 2009.

[4] D.V. Cicchetti. Peer review for manuscript and grants submissions: A cross-disciplinary investigation. Behavioral and Brain Sciences. 14(1), pp. 119–134, 1991.

[5] W.D. Cook, B. Golany, M. Kress, M. Penn, and T. Raviv. Optimal allocation of proposals to reviewers to facilitate effective ranking. Management Science. 51(4), pp. 655-661, 2005.

[6] W.D. Cook. Distance-based and adhoc consensus models in ordinal preference ranking. European Journal of Operational Research. 172, pp. 369–385, 2006.

[7] W. D. Cook, B. Golany, M. Kress, M. Penn, and T. Raviv. Creating a consensus ranking of proposals from reviewers' partial ordinal rankings. Computers and Operations Research, 34(4), pp. 954–965, 2007.

[8] C. Hodgson. How reliable is peer review? An examination of operating grant proposals simultaneously submitted to two similar peer review systems. Journal of Clinical Epidemiology. 50(11), pp. 1189-1195, 1997.

[9] N. S. Iyer, "A family of dominance rules for multiattribute decision making under uncertainty," IEEE Transactions on Systems, Man, and Cybernetics—Part A: Systems and Humans. Vol. 33, no. 4, pp. 441–450, July 2003.

[10] L. Langfeldt. Expert panels evaluating research: decision making and sources of bias. Research Education. 13(1), pp. 51-62, 2004.

[11] B. Malakooti. Ranking and screening multiple criteria alternatives with partial information and use of ordinal and cardinal strength of preferences. IEEE Transactions on Systems, Man, and Cybernetics—Part A: Systems and Humans. Vol. 30, No. 3, pp. 355-568, May 2000.

[12] A. Y. Oruç, N. Yetis, and O. Cebeci. Effective evaluation and funding of research projects. Peer review: Its present and future state. Prague, pp. 74–84, 2006.

[13] K.S. Park, S.H. Kim, and W.C. Yoon. Establishing strict dominance between alternatives with special type of incomplete information. European Journal of Operational Research. 96 (2), pp. 398-406, 1997.

[14] P. Sarabando and L.C. Dias. Multiattribute choice with ordinal Information: A comparison of different decision rules. IEEE Transactions on Systems, Man, and Cybernetics—Part A: Systems and Humans. Vol. 39, No. 3, pp. 545-554, May 2009.

[15] S. Wessely. Peer review of grant applications: what do we know? The Lancet. 352 (9124), pp. 301-305, 1998.

[16] A. Y. Oruç and A. Atmaca. Asymptotically optimal assignments in ordinal evaluations of proposals. arXiv:0908.3233v1. Aug. 2009

[17] A. Y. Oruç and A. Atmaca. On ordinal covering of proposals using balanced incomplete block designs. arXiv:0909.3533v1. Sept. 2009.

TABLE IV. Assignment of 18 documents to 12 experts, each with a capacity of 6.

e_1	d_1	d_2	d_3	d_4	d_5	d_6												
e_2							d_7	d_8	d_9	d_{10}	d_{11}	d_{12}						
e_3													d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}
e_4	d_1	d_2					d_7	d_8					d_{13}	d_{14}				
e_5			d_3	d_4					d_9	d_{10}			d_{13}	d_{14}				
e_6					d_5	d_6					d_{11}	d_{12}	d_{13}	d_{14}				
e_7	d_1	d_2							d_9	d_{10}					d_{15}	d_{16}		
e_8			d_3	d_4							d_{11}	d_{12}			d_{15}	d_{16}		
e_9					d_5	d_6	d_7	d_8							d_{15}	d_{16}		
e_{10}	d_1	d_2									d_{11}	d_{12}					d_{17}	d_{18}
e_{11}			d_3	d_4			d_7	d_8									d_{17}	d_{18}
e_{12}					d_5	d_6			d_9	d_{10}							d_{17}	d_{18}